Fermi Questions

Larry Weinstein, **Column Editor**
Old Dominion University, Norfolk, VA 23529;
weinstein@odu.edu.

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Question 1: The Eiffel Tower

Why does the Eiffel tower appear so airy and why do its models seem so clumsy in comparison to it?

(Guest question and answer by Laura Weiss, University of Geneva)

Fig 1. The Eiffel tower.[1]

Fig 2. An Eiffel tower model.[2]

Answer: DATA: According to Wikipedia, the Eiffel tower is 324 m tall and its steel skeleton weighs 7300 tons.

Let's consider a 30-cm tall metal Eiffel tower model. This is 10^3 times shorter than the original. The "natural" estimate of the model mass is

$$m_{\text{model}} = \frac{h_{\text{model}}}{h_{\text{tower}}} m_{\text{tower}} = (10^{-3})(7\times10^6\,\text{kg}) = 7\times10^3\ \text{kg}$$

Oh, oh, there is a big problem: a 30-cm model cannot weigh 7 tons!

This shows the classical difficulty for the layman (including students) to think in three dimensions when one dimension is salient.

Let's reason a bit further. When one shrinks the object, all three dimensions need to be reduced equally. Thus,

$$m_{\text{model}} = \left(\frac{h_{\text{model}}}{h_{\text{tower}}}\right)^3 m_{\text{tower}} = (10^{-3})^3(7\times10^6\,\text{kg}) = 7\times10^{-3}\ \text{kg}$$

We still have a problem. The model, which couldn't weigh 7 tons, can't weigh 7 grams either! This time, the result is much too small.

What is wrong with this second answer? The Eiffel tower is made of girders whose width is on the order of decimeters and whose thickness is on the order of centimeters. If you scale this width by a thousand, you obtain a girder width of

$$w_{\text{beam}}^{\text{model}} = (10^{-3})(0.1\ \text{m}) = 10^{-4}\,\text{m}.$$

Clearly it is impossible to construct a 30-cm tall durable model with pieces a tenth of a millimeter (100 microns) wide and a hundredth of a millimeter (10 microns) thick! For comparison, aluminum foil is 25 microns thick. That is why Eiffel tower models are always so clumsy, and so much less elegant than their taller original.

Now let's consider the "airiness" of the Eiffel tower. Looking at the pictures we can estimate that the width of the base is about 1/3 the height. Modeling the tower as a pyramid, this means that its volume is

$$V_{\text{tower}} = \frac{1}{3}(10^2\,\text{m})^2(3\times10^2\,\text{m}) = 10^6\,\text{m}^3$$

and its average density is

$$\rho_{\text{tower}} = \frac{m_{\text{tower}}}{V_{\text{tower}}} = \frac{7\times10^6\,\text{kg}}{10^6\,\text{m}^3} = 7\ \text{kg/m}^3.$$

This is 10^{-3} of the density of steel. Thus, 99.9% of the Eiffel tower is empty space. This explains once more why it is so difficult to create graceful scale models!

References

1. http://fr.wikipedia.org/wiki/Tour_Eiffel.
2. Photo Laura Weiss.

Question 2: Supertankers to Japan

How many oil tankers are in transit from the Middle East to Japan and how far apart are the oil tankers?

Answer: In order to estimate this, we need to estimate the amount of oil consumed by Japan, the proportion of oil it imports from the Middle East, the time it takes for one oil tanker to travel from the Middle East to Japan, and the size of an oil tanker. In order to estimate the travel time, we will further need to estimate the distance and the speed. Whew! Let's get started.

In order to estimate Japan's oil consumption, we need to know the population of Japan and the per capita oil consumption. Japan's population is smaller than that of the U.S. (<100%), but not that much smaller (>10%), so we will take the geometric mean and estimate that

$N_J = 0.3 \times N_{US} = 0.3 \times (3 \times 10^8) = 10^8$.

Since oil is used primarily as a transportation fuel, we will estimate the average gasoline consumption. America has about two cars for every three people. At 15,000 miles per year and 20 miles per gallon, each American uses

$$v_{gas}^{U.S.} = \frac{2 \text{ cars}}{3 \text{ people}} \times \frac{1.5 \times 10^4 \text{ miles/yr}}{20 \text{ mpg}} = 500 \text{ gal / person} \cdot \text{yr}.$$

We can double this to reflect commercial transportation and other oil use and then we can halve this to reflect that fact that Japan is more compact than the U.S. and therefore transportation distances are shorter. At 4 liters per gallon, this means that Japan uses

$$V_{oil}^{J} = (10^8 \text{ people})(500 \frac{\text{gal}}{\text{person} \cdot \text{yr}}) = 2 \times 10^{11} \text{ L/yr},$$

or, assuming that oil has the same density as water,

$$M_{oil}^{J} = 2 \times 10^{11} \text{ kg/yr} = 2 \times 10^8 \text{ ton/yr}.$$

A typical supertanker is larger than a supercarrier and displaces more than 2×10^5 tons (and no, it does not matter for estimation purposes whether these are long tons, short tons or metric tons). Ignoring the mass of the ship itself, this means that Japan needs

$$n_{ships} = \frac{2 \times 10^8 \text{ tons/yr}}{2 \times 10^5 \text{ tons/ship}} = 10^3 \text{ ships/yr},$$

or about three supertankers every day. Japan produces little if any oil. If Japan imports between 2/3 and all of its oil from the Middle East, then there are two or three tankers per day sailing to Japan.

Now we need to estimate the length of the journey. The distance from the Middle East to Japan is more than 10^3 miles and less than 2.5×10^4 miles (the circumference of the globe), so we will estimate 5×10^3 miles. The tanker speed is 10 knots (more than 5 and less than 25 knots). Ignoring the 15% difference between nautical and statute miles, it will take a supertanker 500 hours or about 20 days to travel 5×10^3 miles.

This means that there will be about 10^2 tankers traveling to and from Japan at any given time (2–3 tankers per day times 40 day round trip time). At three tankers per day, they will be spaced about 10 hours or 100 miles apart.

That is a LOT of very large ships.