Fermi Questions

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Solutions for Fermi Questions, October 2013

Question 1: Granularity

Which is more granular (quantized), a stream of water or a stream of light? *(Thanks to Gary White of* The Physics Teacher *for suggesting the question.)*

Answer: To answer this, we need to estimate the number of molecules in a stream of water and the number of photons in a stream of light. Let's start with light. We need to estimate the power of the stream of light and the energy of a single photon. A stream of light could be any beam of light from that emitted by a 5 mW laser pointer to that emitted by a car dealer's searchlight. The searchlight is two to three orders of magnitude more powerful than a 100-W light bulb (which consumes 100 W of power but only emits about 5 W of visible light) so we'll estimate that it emits 1 kW of visible light.

We can estimate the energy of a visible photon in a few ways. Visible-light photons do not cause sunburns and therefore are non-ionizing. Since the energy of an energetic chemical reaction is about 1.5 V (this is 10% of the binding energy of hydrogen and equal to the energy a single electron receives from a single chemical reaction in a AA or D battery), the energy of a visible-light photon should be about 1 eV. Alternatively, if you happen to remember Planck's constant and the wavelength of light, you can calculate it:

$$E_\gamma = hc/\lambda = \frac{(7\times10^{-34}\,\mathrm{J\cdot s})(3\times10^{8}\,\mathrm{m/s})}{5\times10^{-7}\,\mathrm{m}}$$
$$= 4\times10^{-19}\,\mathrm{J} = 2\ \mathrm{eV}$$

The number of photons per second in a 1-W stream of light is just the ratio of the stream power and photon energy:

$$\frac{dN_\gamma}{dt} = \frac{1\ \mathrm{W}}{4\times10^{-19}\,\mathrm{J/photon}} = 2\times10^{18}\,\mathrm{photons/s}$$

Divide by a thousand for the laser pointer and multiply by a thousand for the searchlight.

Now let's consider a stream of water such as that flowing from a kitchen faucet or from a fire hose. We will need to estimate the volume flow rate, the density of water, and the number of molecules per gram. We can estimate the water flow rate either by estimating the flow diameter and flow speed or by estimating how long it takes to fill a container. The stream of water from a kitchen faucet is about 1 cm^2 and it flows at more than 1 and less than 10 m/s (between 2 and 20 mph), giving a flow rate of about 300 cm^3/s. Alternatively, it takes about 10 s (more than 1 and fewer than 100 s) to fill a 1-liter container, giving a flow rate of 100 cm^3/s. The density of water is 1 (in units of g/cm^3, kg/L, oz/oz, lb/pint, or tons/m^3, your choice), giving a mass flow rate of 200 g/s.

To calculate the number of molecules in a gram, we need to remember Avogadro's number:

$$n_\mathrm{m} = \frac{6\times10^{23}\,\#/\mathrm{mole}}{18\ \mathrm{g/mole}} = 3\times10^{22}\,\#/\mathrm{g}.$$

The number of molecules per second in the water flowing out of your kitchen faucet is thus

$$\frac{dN_\mathrm{m}}{dt} = (200\ \mathrm{g/s})(3\times10^{22}\,\#/\mathrm{g})$$
$$= 6\times10^{24}\,\#/\mathrm{s}\ .$$

Therefore, a stream of light, whether from a laser pointer or a big searchlight, is much more granular (quantized) than the stream of water from your kitchen faucet, and much much much more granular than the stream of water from a fire hose.

Question 2: Coffee Beans

How many coffee beans do Americans consume yearly? (*Thanks to Chuck Adler of St. Mary's College of Maryland for the question.)*

Answer: In order to estimate this, we need to estimate the proportion of Americans who drink coffee, the number of cups of coffee they each consume, and the number of coffee beans per cup. Coffee drinkers comprise more than 10% and less than 100% of the 3×10^8 Americans, giving an estimate of 30%. These coffee drinkers consume more than one and fewer than 10 cups of coffee per day, giving an estimate of three cups per day. One cup of coffee is made from 1 to 2 tablespoons of coffee beans (that is 15 to 30 ml, for those people ignorant of our gloriously intricate suite of volume measures). One coffee bean has approximate volume (0.5 cm)$\times$(0.5 cm)$\times$(1 cm) = 0.2 cm^3. This means that Americans consume

$$N_{\text{beans}} = \frac{(3\times 10^8\ \text{Am})(0.3)(3\ \text{cc}/\text{Am}\cdot\text{day})(400\ \text{day/yr})(20\ \text{cm}^3/\text{cc})}{0.2\ \text{cm}^3/\text{bean}}$$

$$= 10^{13}\ \text{beans/yr}.$$

Wow. We're really full of beans! If stacked vertically without toppling, that would reach a distance of 10^{13} cm = 10^8 km, or 2/3 of the distance to the Sun. At that point, many of the coffee beans would be quite well roasted.