Fermi Questions

***Larry Weinstein,* Column Editor**
Old Dominion University, Norfolk, VA 23529;
weinstein@odu.edu.

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Question 1: Flatness

Which is flatter, Kansas or a pancake?

Answer: To answer this, we first need to define flatness. Clearly, the tallest terrain feature in Kansas will be much taller than the tallest terrain feature on a pancake. Therefore we will consider the relative flatness, ratio of the height of the tallest terrain feature to the horizontal size.

We'll start by estimating the horizontal size of pancakes and Kansas. A pancake is about 10 cm in diameter (more than 1 cm and less than 1 m). Kansas is about 500 miles (800 km) across (more than 100 miles and less than the size of the United States, or 3×10^3 miles).

The primary terrain features on a pancake are depressions. These are typically 1 to 2 mm deep. The tallest mountain in Kansas is more than 60 ft (the height of Mt. Trashmore in Virginia Beach) and less than 6×10^3 ft (the height of the tallest mountain in the Appalachians) so we will estimate 600 ft or 2×10^2 m. (Note that the actual altitude variation in Kansas is 10^3 m, so we are off by a factor of five.)

This means that the flatness of a pancake is

$$f_{\text{pancake}} = \frac{1\text{ mm}}{10\text{ cm}} = 10^{-2}$$

and the flatness of Kansas is (using the correct altitude variation, rather than our within-a-factor-of-ten estimate)

$$f_{\text{Kansas}} = \frac{10^3\text{m}}{8\times10^5\text{m}} = 10^{-3}$$

Thus Kansas is about 10 times flatter than a pancake.

If we could make perfect pancakes with no depressions, then the surface variation of a pancake would be about 0.1 mm (the thickness of a sheet of paper). In that case, we could accurately claim that a pancake is as flat as Kansas.

Question 2: Falling leaves

What is the potential energy of all the leaves that fall on the United States during the fall?

Answer: In order to estimate this, we need to estimate the mass and the height of the falling leaves. Estimating the height is easy. The average height is more than 1 m and less than 100 m, so we will take the geometric mean and estimate 10 m (30 ft).

Estimating the mass of all the leaves is more difficult. We could estimate the area of the United States, the average tree density, the average number of leaves per tree, and average mass per leaf. Alternatively, we could estimate the number of bags of leaves raked by the average homeowner, the weight of each bag, the area of his or her land, and the area of the United States. On the third hand, we could estimate the area of the United States, the fraction of forested land area, and the average thickness (and density) of the compacted fallen leaves. As a lazy physicist, I'll use the third method.

There is a three-hour time difference between east and west coasts. Thus, the distance between New York and California is about one-eighth the circumference of the globe, or about 3×10^3 mi or 5×10^3 km. The north-south distance is a bit less than half of the east-west distance, or about 2×10^3 km. This gives the area of the contiguous United States

$$A = (2\times10^3\text{ km})(5\times10^3\text{ km}) = 10^7\text{ km}^2 .$$

This is actually only about 20% too high (and is remarkably closer if we include Alaska and Hawaii).

The fraction of this area that is forested is less than 100% and more than 10% so we will estimate 30%, giving a forested area of $A_f = 3\times10^6\text{ km}^2$. (Feel free to correct this estimate for the fraction of forests that are deciduous. We won't bother.)

Now we just need to estimate the thickness and density of the compacted fallen leaves. The thickness will be more than 1 mm and less than 1 cm so we will estimate an average thickness of $t = 3$ mm. If there are about 10 leaves per millimeter, then this corresponds to an average thickness of 30 leaves. Since the leaf density on the ground in the forest will be more than 1 and less than 10^3, 30 is a reasonable estimate.

The density of the leaves is a little less than the density of water, so we will estimate $\rho = 10^3\text{ kg/m}^3$. This means that the total mass of all the leaves will be

$$\begin{aligned} m &= tA_f\rho \\ &= (3\times10^{-3}\text{m})(3\times10^6\text{km}^2)(10^3\text{kg/m}^3) \\ &= 10^{13}\text{kg} = 10^{10}\text{tons} \end{aligned}$$

Wow. That's a lot of leaves. I'm glad that I don't have to rake them all.

The total potential energy of all those leaves will be

$$
\begin{aligned}
KE = mgh &= (10^{13}\ \text{kg})(10\ \text{m/s}^2)(30\ \text{m}) \\
&= 3\times10^{15}\ \text{J},
\end{aligned}
$$

or about one megaton of TNT. This also seems like a lot. Fortunately, this is spread out over a large area and a large time and is dissipated by air resistance. The energy density is only $e = KE / A = 3\times10^{15}$ J / 3×10^{12} m^2 = 10^3 J/m^2 which is equal to one second of the solar energy flux at Earth's orbit.