Fermi Questions

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Solutions for Fermi Questions, May 2013

Question 1: White school bus roofs

Many yellow school buses have white roofs. How much difference does this make? How much fuel would a white roof save, if you are driving an air conditioned bus in the summer?

Answer: To estimate the energy saved in air conditioning by having a white rather than a yellow roof, we need to estimate the area of the roof, the incident solar flux, the change in albedo (reflectivity) of the roof, and the efficiency of the air conditioner. If the bus seats 60 people, then it has 15 rows of seats spaced at about 1 m for a total length of about 20 m. The bus is about 10 feet or 3 m wide, giving a roof area of 60 m^2. The solar flux outside the atmosphere at Earth orbit is about 10^3 W/m^2 and we will estimate that about half that reaches the ground on a sunny day. At the equinox, the angle of incidence of the sunlight on the roof will vary from your latitude (mine is about 40^o) to 90^o. Let's assume the average of $\cos(\theta)$ is 0.5 (this average will be smaller in the winter and larger in the summer). This means that there is about

$$P = (10^3 \mathrm{W/m^2})(0.25)(60\ \mathrm{m^2}) = 2\times10^4\ \mathrm{W}$$

of sunlight incident on the roof of the bus (more in the summer and less in the winter).

Now we need to estimate the change in the albedo. School-bus yellow is a nice bright color so it has a relatively large albedo. If we make a simple-minded model of light with red, green, and blue bands, then yellow reflects red and green and therefore has an albedo of 2/3. White reflects all three colors and therefore has an albedo of 1. The difference in these ideal albedos is 0.3 so that the difference in the real albedos is probably slightly smaller. This means that we will estimate the decreased heat load on the bus due to the increased albedo as

$$\Delta P = (0.2)(2\times10^4\ \mathrm{W}) = 4\times10^3\ \mathrm{W}.$$

This is the equivalent of three portable electric heaters, so it is a significant change in the heat load and presumably in the internal temperature of a non-air-conditioned school bus.

Now let's estimate the fuel savings if the bus is air conditioned. The air conditioner needs to remove this much extra power from the bus:

$$\Delta P = (4\times10^3\ \mathrm{W})\frac{4\times10^3\ \mathrm{s}}{1\ \mathrm{hr}} = 2\times10^7\ \mathrm{J/hr}\ .$$

Air conditioners are heat pumps and can typically remove several times more energy than the work supplied (in thermodynamic terms, the efficiency is about 3 and Q_H is several times larger than W). However, the efficiency of a bus engine for converting the energy in its fuel to work is about ¼ so these efficiencies cancel. Gasoline and diesel fuel each have an energy density of about 3×10^7 J/L, so that the bus will save about one liter of fuel per hour (or about \$1 per hour) if its roof is white. While this is not a lot, the lower the heat load, the smaller the air conditioner needed and the easier it is to maintain a constant internal temperature.

However, I do not recommend running out to spray-paint the roof of your car.

Question 2: Botched operations

A March 2013 *USA Today* article reported that "More than a dozen times a day, doctors sew up patients with sponges and other supplies mistakenly left inside." What is the probability that this will happen during a randomly selected surgical operation?

Answer: We need to estimate the total number of operations performed yearly in the United States. We can estimate this from the supply side, by estimating the number of hospitals, their operating rooms, and the number of operations they can perform or we can estimate this from the demand side, by estimating the (fractional) number of operations an individual will have each year. Let's try both ways and see if they agree.

My metropolitan area has a population of about 10^6 and has about 10 hospitals (more than one and less than 100). Each hospital has about five operating rooms (more than one and less than 20) and each OR can perform five operations per day (more than one and less than 20). At 250 days per year (ignoring weekends), this gives

$$n = (250\ \mathrm{days/yr})(5\ \mathrm{ops/day\cdot OR})(5\ \mathrm{ORs/hosp})(10\ \mathrm{hosp}/10^6\ \mathrm{people}) = 6\times10^{-2}\ \mathrm{ops/person\text{-}yr}.$$

With a population of 3×10^8, this gives

$$N_{ops} = (3\times10^8 \text{ people})(6\times10^{-2} \text{ ops/person-yr}) = 2\times10^7 \text{ ops/yr.}$$

Wow! That is a LOT of operations.

Now let's estimate from the demand side. The average American will have more than one and fewer than 10 operations in his or her 80-year lifetime. That is about one every 30 years (which would be quite enough for me). This implies that

$$N_{ops} = (3\times10^8 \text{ people})(3\times10^{-2} \text{ ops/person-yr}) = 10^7 \text{ ops/yr,}$$

which agrees with our previous estimate.

Estimating the probability that a single operation leaves something behind is now straightforward.

$$P = \frac{N_{\text{incidents}}}{N_{\text{ops}}} = \frac{(12 \text{ inc/day})(400 \text{ day/yr})}{(10^7 \text{ ops/yr})} = 5\times10^{-4}$$

or 1 in 2000. From a personal standpoint, this is a comfortingly small probability. From a societal standpoint, 5×10^3 incidents per year is much less than the 3×10^4 automobile fatalities and much much less than the 2×10^6 automobile injuries each year.

However, one reason for estimation is to decide a course of action. To do that we need to know the costs and benefits of the action. The typical cost of one of these surgical incidents is about \$$10^5$ (more than \$$10^4$ and less than \$$10^6$) giving an average cost of \$50 per operation. Therefore, any solution that costs a lot less than \$50 per operation is worthwhile.