Fermi Questions

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Solutions for Fermi Questions, May 2012

Question 1: Flashy bugs

How many insects hit the DC comics superhero The Flash as he runs at extraordinary speed around the world?

Answer: In order to estimate the number of insects hitting The Flash as he circumnavigates the globe, we need to estimate his frontal area, the distance he travels, and the density of insects. The first two are easy.

The frontal area of a human is 1 m^2. The circumference of the globe is $c = 2\pi R = 4\times10^7$ m. However, since there are many fewer insects over water and oceans cover ¾ of the globe, we will use $c = 10^7$ m. (As we estimated in this month's other question, The Flash definitely runs fast enough to run over the water.)

Now we need to estimate the density of flying insects. Let's consider spring or summer, when the insect density is the greatest. We can estimate this two ways, either by estimating the density directly or by examining the windshields of automobiles. The density of flying insects near ground level in grassy or forested areas will be less than 10 m^{-3} and more than 10^{-1} m^{-3}, giving an estimate of $\rho = 1$ m^{-3}. However, many of these will be swept around The Flash by the wind stream and not hit him, so this is an overestimate.

Automobile windshields hit one insect every few kilometers. This implies an insect density of

$$\rho = \frac{N}{V} = \frac{1}{(3\times10^3\,\mathrm{m})(1\ \mathrm{m}^2)} = 3\times10^{-4}\,\mathrm{m}^{-3},$$

where we estimated the frontal area of the windshield as $A = 1$ m^2. However, highways are inhospitable places for insects and they tend to be swept clear of bugs by the other automobiles, so this is an underestimate.

Let's take the geometric mean of these two estimates and use $\rho = 2\times10^{-2}$ m^{-3}. Note that both of these are temperate-zone estimates. Polar regions will have fewer bugs and tropical regions will have more.

Therefore The Flash will hit

$$N = \rho V = (2\times10^{-2}\,\mathrm{m}^3)(10^7\,\mathrm{m})(1\ \mathrm{m}^2) = 2\times10^5$$

insects as he runs around the world. In other words, every square-centimeter of his body will be covered with about 20 bug-splats. Yuck!

This explains why Superman flies at high altitude or in space when he travels long distances.

(Thanks to William Dunlap of Old Dominion University for pointing out the Saturday Morning Breakfast Cereal comic with this idea. For a graphic picture of the result, see http://www.smbc-comics.com/index.php?db=comics&id=1873#comic)

Question 2: Running on water

The common basilisk (also known as the Jesus Christ lizard) can run across the surface of a body of water. How fast does it need to run to stay on the surface? How fast would a human need to run?

Answer: In order to run across the surface of the water, an animal needs to run fast enough so that, at each step, its feet do not have time to sink into the water. We can estimate this maximum contact time from the downward force exerted by its feet on the water, the mass of water that is accelerated by this force, and the maximum distance its foot can depress the water surface. The speed of the animal is then just its stride length divided by the contact time.

The downward force will be equal to or greater than its weight. Lizards are generally much smaller than cats or dogs so that a medium-sized lizard will have a mass between 0.1 and 1 kg. We will take the geometric mean and estimate that $m_L = 0.3$ kg.

Next we need to estimate the mass of the water that is accelerated by this force. The area will be about the area of the lizard's foot. The depth of the water will be about the width of the foot. The width of the foot will be between 1 and 10 cm, giving an estimate of 3 cm. Assuming a square foot, the volume of water that is accelerated by this force will be 30 cm^3, with a mass of $m_w = 0.03$ kg.

This water will then accelerate at

$$a_w = \frac{m_L g}{m_w} = \frac{(0.3\ \mathrm{kg})(10\ \mathrm{m/s^2})}{0.03\ \mathrm{kg}} = 10^2\,\mathrm{m/s^2}.$$

The maximum distance that the lizard's foot can depress the water surface before sinking into it will be more than 0.1 cm and less than 10 cm, so we will estimate d = 1 cm = 10^{-2} m. At $a_w = 10^2$ m/s^2, this implies a maximum contact time of

$$t_{\max} = \sqrt{2d / a_w} = \sqrt{\frac{2\times10^{-2}\,\text{m}}{10^2\,\text{m/s}^2}} \approx 10^{-2}\,\text{s}.$$

Lizards have short legs so that their step length will be only about 3 cm. This gives the minimum speed to run across the water as

$$v_{\min} = \frac{d_{\text{step}}}{t_{\max}} = \frac{3\times10^{-2}\,\text{m}}{10^{-2}\,\text{s}} = 3\ \text{m/s},$$

or about 7 mph. This should be attainable for short periods of time, even by a short-legged lizard.

Now let's consider the minimum human velocity for water running. Let's assume the usual human mass, in round numbers, of 100 kg. Our feet are about 0.1 m by 0.3 m so that the mass of the water displaced at each step is

$$m_w = \rho_w V = (10^3\,\text{kg/m}^3)(0.1\ \text{m})^2(0.3\ \text{m}) = 3\ \text{kg},$$

where we assumed that the depth of the displaced water equals the width of our feet. Feel free to adjust these numbers for your mass and foot size.

Now the water will accelerate at

$$a_w = \frac{m_L g}{m_w} = \frac{(100\ \text{kg})(10\ \text{m/s}^2)}{3\ \text{kg}} = 3\times10^2\,\text{m/s}^2,$$

so that the maximum contact time per step is

$$t_{\max} = \sqrt{2d / a_w} = \sqrt{\frac{2\times10^{-2}\,\text{m}}{3\times10^2\,\text{m/s}^2}} = 10^{-2}\,\text{s},$$

which is the same as the lizard's.

However, our step length must be at least 0.3 m. This is because our step length must be longer than our foot length so at each step our foot lands on undisturbed water. This means that we humans need a velocity of at least

$$v_{\min} = \frac{d_{\text{step}}}{t_{\max}} = \frac{3\times10^{-1}\,\text{m}}{10^{-2}\,\text{s}} = 30\ \text{m/s},$$

or about 70 mph.

If our feet were 10 times larger, this would increase the displaced water mass by a factor of 30, increase the time per step by a factor of $\sqrt{30} \approx 5$, and decrease the minimum velocity by the same factor to $v_{\min}$ = 6 m/s or about 12 mph. However, it will be very difficult to run that fast on water (or any other surface) wearing the equivalent of very large snowshoes.

Alas, we will need to leave water running to the lizards.