Fermi Questions

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Question 1: Loading ships

When a passenger boards a boat or a ship, how much does the vessel's draft increase? Consider canoes and cruise ships.

Answer: To estimate this, we need to know the mass of the passenger and the density of water and then we need to estimate the bottom surface area of the vessel. We'll consider a typical passenger with a mass (including luggage) of 100 kg and typical water with a density of 10^3 kg/m^3.

A typical three- or four-person canoe is 17 ft (5 m) long and about 3 ft (1 m) wide (more than 1 and less than 10 ft) at its widest point. This gives a bottom surface area at the water line of $A = (1/2)lw = 2$ m^2.

The 100-kg passenger will cause the canoe to displace an additional 0.1 m^3 of water. Thus, the added depth will be

$$d = \frac{V}{A} = \frac{0.1 \text{ m}^3}{2 \text{ m}^2} = 5\times 10^{-2}\text{m}$$
$$= 5 \text{ cm},$$

which is quite noticeable in a canoe.

A cruise ship is slightly larger than a canoe and even larger than a football field. Its length is more than 100 m and less than 10^3 m, giving an estimate of $l = 3\times10^2$ m or about 60 c (60 canoes). Its aspect ratio is about the same as a canoe, giving a width (in round numbers) of 10^2 m. This gives a bottom surface area of

$$A = (1/2)lw = (0.5)(3\times10^2\text{m})(10^2\text{m})$$
$$= 2\times10^4\text{m}^2.$$

The added depth in this case will be

$$d = \frac{V}{A} = \frac{0.1 \text{ m}^3}{2\times10^4 \text{ m}^2} = 5\times10^{-6}\text{m}$$
$$= 5\ \mu\text{m},$$

which will be completely unnoticeable. Even the addition of all 4000 passengers will only increase the depth by 2 cm.

Question 2: Ship fuel

How much extra fuel is needed to transport one passenger across the Atlantic Ocean on a cruise ship?

Answer: In order to estimate this, we need to estimate the distance traveled, the speed of the ship, and the drag force of the water. The distance is comparable to but further than the distance from New York to LA (5000 km), so we will estimate that it is 8000 km. Alternatively, the 5-hour time difference between Eastern Standard Time and Greenwich Mean Time means that the distance is 5/24 of the Earth's circumference or 8000 km (ignoring the factor of $\cos\theta \approx 0.7$ due to latitude). Cruise ships cruise, so we'll estimate their speed as between 1 and 20 m/s, or 5 m/s.

Now we need to figure out the drag force of the water. There are two approaches. If we remember the drag force of fluids as

$$F = \frac{1}{2}C\rho A v^2,$$

then the energy needed to cross the Atlantic is

$$E = Fd = \frac{1}{2}C\rho A d v^2$$

and we just need to estimate the drag constant C. Alternatively, we can start from first principles.

To move through the water, the ship of cross-sectional area A needs to displace water at a rate $\dot{m} = \rho A v$, where $\rho = 10^3$ kg/m^3 is the density of water. To do that, it needs to give the displaced water a transverse velocity $v_T = vw/l$, where l is the length of the ship and w is its width (assuming a diamond-shaped ship). The power needed is then the kinetic energy imparted to the displaced water, or

$$P = \frac{1}{2}\dot{m}{v_T}^2 = \frac{1}{2}\rho A v(vw/l)^2$$
$$= \frac{1}{2}\rho A v^3 (w/l)^2.$$

The total energy needed to travel a distance d is

$$E = Pt = P\frac{d}{v}$$
$$= \frac{1}{2}\rho A d v^2 (w/l)^2.$$

This is the same as the previous estimate (whew!), with the added benefit of directly estimating the drag coefficient, C. Now we just need to estimate the numbers.

In the previous question we estimated the cruise ship's beam to length ratio $w/l = 0.3$ and the additional depth due to one passenger as $\Delta y = 5 \times 10^{-6}$ m. That added depth times the width of the cruise ship gives the additional area due to one passenger,

$$\Delta A = w\Delta y = (10^2\,\mathrm{m})(5\times10^{-6}\,\mathrm{m})$$
$$= 5\times10^{-4}\,\mathrm{m}^2.$$

(This means that the volume of water pushed out of the way by that extra passenger is

$$\Delta V = d\Delta A = (8\times10^6\,\mathrm{m})(5\times10^{-4}\,\mathrm{m}^2)$$
$$= 4\times10^3\,\mathrm{m}^3.$$

That 100-kg passenger pushes 4 tons of water out of his way. That seems somewhat inconsiderate.)

We can now calculate the extra energy to transport one passenger across the Atlantic as

$$\Delta E = 0.5(10^3\,\mathrm{kg/m^3})(5\times10^{-4}\,\mathrm{m}^2)(8\times10^6\,\mathrm{m})(5\mathrm{m/s})^2(0.3)^2$$
$$= 5\times10^6\,\mathrm{J}.$$

Including a factor of four for engine efficiency, the chemical energy needed from fuel is

$$\Delta E_{\mathrm{fuel}} = 2\times10^7\ \mathrm{J},$$

which happens to be the energy contained in one liter of gasoline or diesel fuel. Even if the ship travels at 10 m/s (20 knots), this would only increase to the energy contained in four liters or one gallon.

Wow. That is remarkably little fuel. It is 100 times less than the energy needed to fly that passenger across the Atlantic (see *Guesstimation 2.0*). On the other hand, it explains how oil can be shipped across oceans without burning it all en route.