Fermi Questions

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Question 1: Cool threads

What is the total length of all the thread used to make the clothing you are wearing? How long would it take to spin this thread by hand using a spinning wheel?

Answer: We will break this down into the length of thread needed to make a square meter of cloth and the total area of cloth needed to clothe us. Cloth is made by weaving thread (and only knit-wits will mention alternate ways of making cloth). Fine cloth has more than 10 and less than 10^3 threads per inch, giving an estimate of 10^2 threads per inch or 4×10^3 threads per meter. Each of those threads is 1 m long, so that 1 m^2 of cloth contains 4 km of warp threads and 4 km of weft threads (so cloth is made at warp speed) for a total of 8 km. (Coarsely woven cloth like jeans have only about 25 threads per inch, giving a total length of only 2 km of thread per square meter.)

Our surface area (and hence the area of our clothing) is more than 1 and less than 10 m^2, giving a total of A = 3 m^2. Rounding up to include underwear, our fine clothing contains about 40 km of thread and our coarse clothing contains a mere 10 km.

Treadle-powered spinning wheels, although a tremendous technological advance over the older hand spindles, are still relatively slow. Based on watching historical re-enactors at places like Old Sturbridge Village or Colonial Williamsburg, we can estimate the spinning rate as more than 0.1 and less than 10 cm per second, giving a speed of 1 cm/s or 10^{-2} m/s. At this rate, it would take 4×10^6 s to spin 4×10^4 of thread. Neglecting underwear and only making coarse "home-spun" would still require 6×10^5 s to spin 6 km of thread. That is almost 200 hours of work (five 40-hour weeks) to make the thread for one outfit.

No wonder clothing used to be so expensive!

Question 2: Huricane Sandy

What was the total energy of Hurricane Sandy?

Answer: A hurricane's energy consists of the kinetic energy of its winds and the potential energy of the water stored in its clouds. We will neglect the potential energy of its storm surge because that only occurs as the hurricane nears land.

To estimate the kinetic energy of the winds, we need the wind speed, the wind area, the wind height, and the air density. Hurricane Sandy was a huge storm and had hurricane strength winds ($v > 76$ mph ≈ 35 m/s) extending over a radius of about 50 km and had tropical storm strength winds ($v > 35$ mph ≈ 15 m/s) extending over twice the radius or four times the area. The wind height was more than one and less than 10 km, so we will estimate 3 km. The air density was $\rho = 10^{-3}$ kg/m^3 (gases typically are about 10^3 times less dense than liquids). This gives the volume of air moving at hurricane speeds as

$$V_{hurr} = A_{hurr}h = \pi(5\times10^4 \text{ m})^2(3\times10^3 \text{ m})$$
$$= 2\times10^{13} \text{ m}^3$$

and the volume of air moving at tropical storm speeds as four times that.

Now the kinetic energy of the hurricane wind was

$$KE_{hurr} = \frac{1}{2}(\rho V_{hurr})v_{hurr}^{\ 2}$$
$$= \frac{1}{2}(1 \text{ kg/m}^3)(2\times10^{13}\text{m}^3)(35 \text{ m/s})^2$$
$$= 10^{16} \text{ J}$$

The kinetic energy of the tropical storm wind was also about 10^{16} J, because the air volume was four times greater and the air speed was two times smaller so that the total kinetic energy is $KE = 2\times10^{16}$ J or about 5 megatons of TNT.

We can estimate the potential energy of the water in the clouds either by estimating the proportion of water in the air or the total rainfall. The second method is easier. The rainfall from the hurricane was more than 0.01 and less than 1 m or about 0.1 m. This rain fell over an area of

$$A_{rain} = \pi\times(10^5 \text{ m})^2$$
$$= 3\times10^{10} \text{ m}^2$$

This means that the mass of the rain was

$$m_{rain} = \rho A t$$
$$= (10^3 \text{ kg/m}^3)(3\times10^{10}\text{m}^2)(0.1 \text{ m})$$
$$= 3\times10^{12} \text{ kg}$$

If the rain fell from 3 km up (more than 1 km and less than 10 km), then its potential energy was:

$$PE = m_{rain}\,gh$$

$$= (3\times10^{12}\ \text{kg})(10\ \text{N/kg})(3\times10^{3}\ \text{m}),$$
$$= 10^{17}\ \text{J},$$

or about 25 megatons of TNT. Wow! There was more energy in the potential energy of the rain than in the kinetic energy of the wind!

Since the rain by itself did little impact damage, this seems much too large. Let's instead consider the impact energy of the rain. Rain drops are slowed by air resistance and hit the ground at about 20 m/s (your sedan's back window is not hit by rain drops when traveling at speeds greater than about 20 m/s or 45 mph) so the kinetic energy of this rain as it hits the ground was

$$KE_{\text{rain}} = \frac{1}{2}mv^2$$
$$= 0.5(3\times10^{12}\ \text{kg})(20\ \text{m/s})^2$$
$$= 6\times10^{14}\ \text{J}.$$

While the potential energy of the water was several times larger than the kinetic energy of the wind, the impact energy of the rain was about 100 times smaller. This makes much more sense.

Thus, the effective energy of a hurricane is about 5 megatons of TNT. That is still rather a lot.