Fermi Questions

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Question 1: Trash to fuel

In the movie "Back to the Future," the protagonists fuel their "car" with trash. If our cars could burn trash, what fraction of our transportation fuel needs could be supplied? (Thanks to Prof. Hyde's "Physics on the Back of an Envelope" class at Old Dominion University for the question.)

Answer: In order to answer this, we need to estimate the energy consumed by our cars and the energy available in our trash. Since there is about one car per person in the United States, we can consider the trash generated and fuel consumed per person. A typical American drives 10 to 15 thousand miles per year in a car that gets 20 to 30 miles per gallon and therefore consumes 500 gallons (2×10^3 L) of gasoline per year with a mass of about 2×10^3 kg. That same American generates about 10 to 20 pounds of trash per week, or about 10^3 pounds (500 kg) of trash per year.

Wow! We burn much more gasoline than we generate trash.

Gasoline is composed of hydrocarbon chains $H(CH_2)_n H$, where n is the length of the chain. Trash is composed mostly of paper, food scraps, and plastic, with some noncombustible glass and metal. The energy density of paper is more than 10% and less than 100% of gasoline's so we will estimate 30%. The energy density of food scraps is less than that and the energy density of plastic is more than that so we will estimate that the overall energy density of trash is 30% that of gasoline.

Thus, since trash production is one-fourth of gasoline consumption and since the energy density of trash is about one-third that of gasoline, even if we could burn trash in our cars as efficiently as we burn gasoline, we would only reduce our gasoline consumption by 10%.

However, this is only true if we consider chemical energy. If we could convert trash to energy using nuclear fusion (i.e., by fusing hydrogen nuclei to helium), then we would produce a LOT more energy. Trash is about 10% hydrogen by mass (since there is about one hydrogen for every carbon, nitrogen, or oxygen, or, alternatively, it is more than 1% and less than 100% hydrogen). When fused to helium, about 0.1% of the hydrogen mass is converted to energy. This means that the 500 kg per person per year of trash will provide

$$E = mc^2$$
$$= (500\text{ kg})(0.1)(10^{-3})(3\times10^8\text{ m/s})^2$$
$$= 5\times10^{15}\text{ J}.$$

This equals the chemical energy contained in

$$V_{\text{gas}} = \frac{5\times10^{15}\text{ J}}{3\times10^{7}\text{ J/L}} = 2\times10^{8}\text{ L}.$$

Unfortunately, I doubt we will be fusing hydrogen into helium in our automobile engines any time soon (no, not even in a DeLorean).

Question 2: Planes from cans

We consume and presumably recycle many cans of soda, juice, and beer on a typical commercial airplane flight. How many flights would it take to collect enough soda cans to build a new airplane?

Answer: To answer this we need to estimate the mass of the aluminum air-frame of a commercial airplane and the mass of aluminum recycled on each flight. Rather than trying to estimate the overall mass of an airplane, let's estimate the per-passenger mass by comparing to better-known vehicles. A light passenger automobile has a mass of about one ton. An airplane has an aluminum body to reduce weight, but it also needs much more mass (aka: "wings") to provide lift. Therefore, we will estimate that the per-passenger mass of a commercial airplane is also about one ton (it's certainly more than 10^2 kg and less than 10^4 kg). The aluminum air frame comprises more than 10% and less than 100% of the airplane mass, so we will estimate 30%, or 300 kg per person.

On a typical short airplane flight, the average passenger is graciously provided with a munificent half a can of liquid. A typical soda can has more mass than a paper clip (1 g) and less mass than 100 cm^3 of water, so we will estimate 10 g or 10^{-2} kg. In order to use round numbers, we will profligately double our estimate of liquid consumed per passenger to the contents of one entire can, or 10 g of aluminum.

Thus, the number of flights needed to accumulate enough aluminum to build a new airplane is

$$N = \frac{300\ \text{kg}}{10^{-2}\ \text{kg/flight}}$$
$$= 3 \times 10^4\ \text{flights.}$$

At three flights per day (round numbers!), this will require 10^4 days or 30 years. At 10 flights per day, it will require a mere 10 years.