Fermi Questions

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Question 1: The dinosaur meteoroid and the day

There is strong evidence that the impact of a 10-km meteoroid killed all the dinosaurs except for the ancestors of the canary. By how much could this meteoroid have changed the length of the day (i.e., the rotational period of the Earth)?

Answer: To estimate the change in the length of the day, we need to estimate the maximum fractional change that the meteoroid could make to the Earth's angular momentum. To do that, we'll need to estimate the maximum angular momentum that the meteoroid could have (with respect to the center of the Earth) and compare that to the Earth's angular momentum. To estimate the angular momentum, $L = mvr$, we need to find the mass and speed of the meteoroid. Since both the meteoroid and the Earth are orbiting the Sun, both will have about the same speed. (You can also calculate the meteoroid's speed by assuming that it falls in from infinity in the Sun's gravitational well. If you do this, you will get a speed that is $\sqrt{2}$ times the Earth's orbital speed. This is because escape velocity is $\sqrt{2}$ times circular orbit speed. Anyway, we'll ignore this factor of 1.4.) The Earth's orbital speed is[1]

$$v = \frac{2\pi R_{ES}}{T_{year}} = \frac{2\pi(1.5\times10^{11}\,\text{m})}{\pi\times10^{7}\,\text{s}} = 3\times10^{4}\,\text{m/s}.$$

Escape velocity from the Earth is three times smaller (11 km/s) and thus the Earth's gravitational field will have a negligible effect on the meteoroid's speed of impact. Because the impact speed could be anywhere between a value that is much less than v (a following collision) and $2v$ (a head-on collision), we will use v.

We will use choose a rocky meteoroid, so its density will be somewhere between water (1) and iron (8 tons/m^3) with a geometric mean of 3 tons/m^3. Thus, the mass of the meteoroid is

$$m = \rho V = 5\cdot10^3\ \text{kg/m}^3 \times (10^4\ \text{m})^3 = 5\cdot10^{15}\ \text{kg}.$$

The maximum angular momentum change, ΔL, due to the meteoroid will occur if it hits the Earth tangentially so that $r = r_{Earth} = 6\cdot10^6$ m. The Earth's angular momentum is $L = I\omega \approx Mr^2\omega$ [ignoring the pesky factor of 0.4 for the moment of inertia of a sphere (which only physics professors will remember)] where $\omega = 2\pi / T_{day}$.

The fractional change in the Earth's day will be:

$$\begin{aligned} f &= \frac{\Delta L}{L} = \frac{2mvr}{Mr^2\omega} \\ &= \frac{2\rho V\, 2\pi R_{ES}/T_{year}}{Mr2\pi/T_{day}} = \frac{2\rho V R_{ES} T_{day}}{Mr T_{year}} \\ &= \frac{2\times(5\cdot10^3\,\text{kg/m}^3)\times(10^4\,\text{m})^3\times(1.5\cdot10^{11}\,\text{m})}{365\times(6\cdot10^{24}\,\text{kg})\times(6\cdot10^6\,\text{m})} \\ &= 10^{-7}. \end{aligned}$$

Thus, the 10-km meteoroid, despite being massive enough to kill most of the life on Earth, could only change the length of the day by about 1 part in 10^7.

1. It is handy to remember that 1 year $\approx \pi\ 10^7$ s. I often tell my students that the factor of π is there because the Earth orbits the Sun in a circle… Actually, the π is just a coincidence, but it is a good test to see if any students are thinking for themselves.

Question 2: School guards

In the wake of school shootings, it is proposed that armed guards be stationed in American schools. How much would this proposal cost for each life that would be saved?

Answer: In order to answer this, we need to estimate the cost of the armed guards and the number of lives that this would save. Note that while we can dispassionately estimate the costs in terms of dollars and the benefits in terms of lives, we cannot determine the value in dollars of saving a life.

At one guard per school, the number of guards needed equals the number of schools which in turn depends on the number of students. The 3×10^8 Americans live an average of 80 years each. We spend 13 of those 80 years in school, from kindergarten through high school, so that the number of K-12 children in the U.S. is

$$N_c = \left(\frac{13}{80}\right)(3\times10^8) = 5\times10^7.$$

The typical school has more than 100 and fewer than 2000 students, giving a geometric mean of 500 students. At 500 students per school, the 5×10^7 students require 10^5 schools and thus 10^5 guards.

The guards will be paid more than \$10 and less than \$50 per hour, so we will estimate \$20, giving a yearly salary of \$$4\times10^4$ (ignoring details like fringe benefits, taxes, and summer vacation). This means that the yearly total cost of these armed guards will be about

$$C = (10^5 \text{ guards})(\$4\times10^4/\text{guard}) = \$4\times10^9.$$

That is a LOT of money. However, it is only \$10 per person or about 0.05% of our gross domestic product.

Now let's estimate the benefits. Let's optimistically assume that the presence of armed guards will deter all mass murderers. The December massacre in Connecticut of over 20 innocent children and teachers was horrible. Fortunately, tragedies like that do not happen often. Let's overestimate the number of deaths at 20 per year. In this case, the cost per life saved will be

$$c = \frac{4\times10^9(\$/\text{year})}{20 \text{ lives/year}} = 2\times10^8(\$/\text{life}).$$

That is still a lot of money. It is 100 times the lifetime earnings of the average American [$(5\times10^4$ \$/yr) $\times$ (40 yr) = \$$2\times10^6$].

Is it worth it? Is this an appropriate policy? Those are moral and political questions that estimation cannot address.